5. A. S. Sukomel, F. F. Tsvetkov, and R. V. Kerimov, Heat Exchange and Hydraulic Resistance in Gas Suspension Motion in Tubes [in Russian], Énergiya, Moscow (1977), p. 190.
6. I. O. Hinze, Prog. Heat Mass Transfer, 6, 433-451 (1971).
7. A. N. Kolmogorov, "Equations of turbulent motion of an incompressible fluid," Izv. Akad. Nauk SSSR, Ser. Fiz., 6, Nos. 1-2, 56-58 (1942).
8. L. Prandtl and K. Wieghardt, "Über ein neues Formelsystem für die ausgebildete Turbulense," Nach. Akad. Wiss. Gottingen Math. Phys., 1, 6-19 (1945).
9. S. Sow, Hydrodynamics of Multiphase Systems [Russian translation], Nauka, Moscow (1971), p. 535.
10. E. P. Mednikov, Turbulent Transfer and Aerosol Precipitation [in Russian], Nauka, Moscow (1981), p. 173.
11. G. Jepson, A. Poll, and W. Smith, "Heat transfer from gas to wall in a gas/solid transport line," Trans. Inst. Chem. Eng., 41, 207-211 (1963).
12. A. Zelnik, "Study of convective heat exchange in flows of a gas containing solid particles," Author's Abstract of Candidate's Dissertation, Moscow (1958).
13. D. Stockburger, "Ser Wärmeaustusch zwischen einer Rohrwand und einem turbulent strömenden Gas-Feststoff Gemis," in: VDI-Forschungsheft 518, VDI-Verlag, Düsseldorf (1966), p. 38.
14. A. S. Sukomel and R. V. Kerimov, "Methods of generalizing experimental data on heat exchange in turbulent motion of gas suspensions in tubes," Tr. Mosk. Energ. Inst., No. 81, 20-26 (1971).
15. L. M. Mirzoeva, "Study of the process of heat liberation of a two-phase flow in a vertical tube," Author's Abstract of Candidate's Dissertation, Baku (1959).
16. L. Farbar and C. A. Depew, "Heat transfer effects to gas—solid mixtures using solid spherical particles of uniform size," Ind. Eng. Chem. Fund., 2, No. 2, 130-135 (1963).

WAVE STRUCTURE OF TURBULENT FLOW IN A TUBE

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The wave theory of turbulence is applied to determine the fluctuation field u of shear flow in a tube.

The particle-wave representation of turbulence reflects several of its quantum-mechanical properties: the fluctuation fields of vortices are manifested as a fluctuation probability wave, encompassing the region of the statistically coupled vortex state. The wavelength of the probability standing wave determines the largest vortex size occurring in the transverse flow scale. A mean (regular) shear flow is realized within the limits of this wave. In the shear model the fluctuation field is represented by means of the wave function ψ, determining the probability wave amplitude — the fluctuation intensity, as well as their linear scales inversely proportional to the wave number. The system of equations and the foundations of the method discussed were published earlier in [1, 2]. Several assumptions on the quantum analogies of turbulence were discussed in [3].

Turbulent flow in a tube at a sufficient distance from its input cross section is realized without longitudinal variation of the fluctuation field. In this case

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2\rho}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{1}{y}\frac{\partial\psi}{\partial y}\right), \quad i = \sqrt{-1}. \tag{1}$$

The behavior of the ψ-wave, described by Eq. (1), is different near the wall ($y \to R$) and near the flow axis ($y \to 0$), since the vortex structure is inhomogeneous in these two regions. The increase in fluctuation intensity at the wall, along with enhanced tendency toward vortex formation, implies existence of an inhomogeneous wave at the walls. The stabilized structure of vortices "torn" from the wall is characteristic of the central flow region, in which the fluctuation level is also stable. The inhomogeneous wave corresponds to the special representation $\psi = a\exp(ib)$, so that for stationary turbulence ($\partial a^2/\partial t = 0$) we obtain, according

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to Eq. (1),

$$h\omega = \frac{1}{2}\rho U^2 - \frac{h^2}{2\rho a}\left(\frac{\partial^2 a}{\partial y^2} + \frac{1}{y}\frac{\partial a}{\partial y}\right), \quad (2)$$

$$\frac{\partial}{\partial y}\left(a^2 y \frac{\partial b}{\partial y}\right) = 0, \quad (3)$$

where $\rho U = h\partial b/\partial y$, $\omega = -\partial b/\partial t$. Characterizing, as usual, the tendency to vortex fluctuation by the probability density a^2, we assume that the fluctuating energy component [the second term in the right-hand side of Eq. (2)] equals $a^2\rho U^2/2$. Consequently:

$$\frac{d^2 a}{d\varphi^2} + \frac{1}{\varphi}\frac{da}{d\varphi} + \left(\frac{db}{d\varphi}\right)^2 a^3 = 0, \quad (4)$$

$$\frac{d}{d\varphi}\left(a^2\varphi\frac{db}{d\varphi}\right) = 0, \quad (5)$$

where $\varphi = y/R$ is the relative radial distance. It is seen from Eq. (5) that

$$\varphi a^2 \frac{db}{d\varphi} = c, \quad (6)$$

where c is an integration constant. Since $db/d\varphi \sim \varphi^{-1}$, the inhomogeneous wave cannot reach the stream axis $\varphi = 0$: the value $U \sim db/d\varphi$ must remain finite everywhere. Covering adjacent vortices to the tube walls, the inhomogeneous region must be a half wavelength, restricted by two nodes, one of which is located at the point $\varphi = 1$, and the other, the internal one, coincides with the boundary $\varphi = \varphi_1$ of the central stabilized wave region of constant intensity. From Eqs. (4), (6) we obtain for the region $\varphi_1 \leqslant \varphi \leqslant 1$ a solution satisfying the conditions $b = 0$, $a = a_0$ at $\varphi = 1$:

$$\varphi = \exp\left[-\frac{a_0}{c}\sqrt{\frac{\pi}{2}}\,\Phi(r)\right], \quad \Phi(r) = \frac{2}{\sqrt{\pi}}\int_0^r \exp(-r^2)\,dr, \quad (7)$$

$$b = \frac{\sqrt{2}}{a_0} D(r), \quad D(r) = \int_0^r \exp r^2\,dr. \quad (8)$$

Here $r^2 = \ln(a_0/a)$, and a_0 is the maximum value of the wave amplitude at the wall $y = R$, where the turbulence intensity u'/U is highest. We recall that $a = u'/U$, where u' is the velocity fluctuation [2, 4]. We use the normalization condition

$$\int_0^{b_0} a^2 db = 1 \quad (9)$$

(the vortex is necessarily found at some phase at the wave). The upper limit b_0 in Eq. (9) corresponds to $\varphi = 0$.

Taking into account that $a^2 db = \sqrt{2}a_0 \exp(-r^2)dr$ for $\varphi_1 \leqslant \varphi \leqslant 1$, as a result of integrating (9) over the tube radius we obtain the relation

$$a_1^2 b_1 + \sqrt{\frac{\pi}{2}}\,a_0\Phi(r_1) = 1, \quad (10)$$

where a_1 = const is the wave intensity for $0 \leqslant \varphi \leqslant \varphi_1$, b_1 is the total variation of its phase in this zone, and the r_1 value corresponds to $a_1 = a_0\exp(-r_1^2)$ and is achieved at the point $\varphi = \varphi_1$. Since in the boundary-layer wave the phase variation is π (the vortex is closed between the two nodes), then, according to Eq. (8):

$$a_0 = \frac{\sqrt{2}}{\pi} D(r_1). \quad (11)$$

To determine b_1 it is necessary to know the shape of the function $b(\varphi)$ in the region $0 \leqslant \varphi \leqslant \varphi_1$. The magnitude of the vortex circulation h/ρ in a similar monochromatic wave region is unchanged — this rule of vortex stabilization was already earlier stated in [4, 5]. In this case the wave number is proportional to the flow velocity, i.e., $U \sim db/d\varphi$. From the coincidence condition of the wave number values at the junction of the two wave regions, the boundary layer and the central one, we obtain for $\varphi \leqslant \varphi_1$ taking account of (6)

$$\frac{db}{d\varphi} = \frac{c}{\varphi_1 a_1^2} \frac{U}{U_1}, \qquad (12)$$

where U_1 is the flow velocity at the boundary layer of the wave $\varphi=\varphi_1$. Consequently,

$$b_1 = \frac{c}{\varphi_1 a_1^2} \int_0^{\varphi_1} \frac{U}{U_1} d\varphi, \qquad (13)$$

while for further analysis it is necessary to determine the regular motion. First one must explain the conditions of the conjugate inhomogeneous and monochromatic waves at the node $\varphi = \varphi_1$, reaching it from the side $\varphi < \varphi_1$. In the region $0 \leqslant \varphi \leqslant \varphi_1$, where $a = a_1 = \text{const}$, we obtain from Eq. (1) for the wave function $\psi = \psi_0 \exp(-i\omega t)$ (ψ_0 is a function of coordinates)

$$\frac{h^2}{2\rho}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{1}{y}\frac{\partial\psi}{\partial y}\right) + \omega h\psi = 0.$$

For $h\omega = (1 + a^2)\rho U^2/2$ and taking account of (12) and the relation $h = \rho U(\partial b/\partial y)^{-1}$ this equation is written in the form

$$z^2 \frac{d^2\psi}{dz^2} + z\frac{d\psi}{dz} + \frac{c^2}{a_1^4}(1 + a_1^2)\frac{F^2}{F_1^2} z^2\psi = 0, \quad \psi=\psi(z), z=\frac{\varphi}{\varphi_1}, \qquad (14)$$

where henceforth $F = U/U_+$, with U_+ being the velocity at the tube axis, and $F_1 = U_1/U_+$. Since φ_1 is a node of the monochromatic wave at which $\psi = 0$ (an otherwise adjoining inhomogeneous and monochromatic wave cannot exist simultaneously), z = 1 corresponds to one of the roots of the function $\psi(z)$. The sharp increase in the velocity U near the wall, where it is maximum, makes it possible to put $F \simeq 1$ in Eq. (14). In this case we have the condition

$$\psi = J_0\left(\frac{c}{a_1^2}\sqrt{1 + a_1^2}\,\frac{z}{F_1}\right). \qquad (15)$$

The zeros of the Bessel function J_0 are $\pi(n - 1/4)$, where n = 1, 2, 3,.... Consequently, for z = 1 we obtain from (15)

$$c = \frac{\pi a_1^2 F_1}{\sqrt{1 + a_1^2}}\left(n - \frac{1}{4}\right), \quad n = 1, 2, 3, \ldots \qquad (16)$$

Vortex observations [6] justify the assumption that upon attachment to the wall a vortex is manifested in the form of a vortex tube which, expanding and narrowing, breaks from the well and tends to the central monochromatic flow region. The displacement of the vortex tube in the transverse flow direction after its break is accompanied by rotation of the vortex contour plane, so that, being removed from the wall, it expands along the flow. The portion of space in which the separating vortex tube is capable of turning in the flow direction is restricted by the large size of the vortex contour at the wall, so that along with them also exist uniform "associated" contours. The maximum achieved diameter of vortex contour, whose plane after separation is oriented normally to the wall, can encompass all distances from the breakup site of the associated vortex to the tube axis. Taking into account that the quantity h/ρ is a circulation, identical for all vortex contours, forming a vortex tube separating from the wall, we obtain its moment of rotation of the order of $M \sim y^2hL$, where L is the large vortex size.

During the break of a vortex tube some portion of its initial angular momentum is "left over" (lost). Since the separation effect is essentially a fluctuation, a statistical measure of the variation in rotation is determined by the fluctuation probability a^2. Therefore, the probable loss is the angular momentum fluctuation of the tube upon separation a^2y^2hL. To conserve the total angular momentum this fluctuation must coincide with the angular momentum of the remaining portion of the vortex tube. At the "root" of the tube, adjacent to the wall, the plane of the vortex contour is parallel to the wall. The angular momentum of this associated portion of the tube, having a height $R - y$, is characterized by the quantity $h(R - y)L^2$.

Thus, we obtain $h(R - y)L^2 = ha^2y^2L$. Consequently,

$$L = \frac{a^2y^2}{R - y} = \frac{a^2\varphi^2}{1 - \varphi} R. \qquad (17)$$

All vortices have this size as long as they are found in the zone of an inhomogeneous boundary-layer half-wavelenth. In the monochromatic wave region vortices occur with size L, acquired by them at the boundary of the inhomogeneous region $\varphi=\varphi_1$. Starting from estimates of the variation of vortex circulation due to velocity fluctuations, the following relation was obtained [4]

$$a^2 L \sqrt{\tau/\rho} = \xi_1 l^2 \partial U/\partial y, \tag{18}$$

in which the matching coefficient ξ_1 is determined by one of the boundary conditions, and l is the fluctuation scale:

$$l = \frac{1}{2}\left|\frac{\partial b}{\partial y}\right|^{-1}(1+a^2)^{-1}. \tag{19}$$

For tube flow the nature of variation of shear stress is known: $\tau=\tau_0\varphi$. Taking into account Eqs. (6), (17), we obtain from (18) an equation for $F = U/U_+$:

$$\frac{dF}{d\varphi} = -\xi\frac{\sqrt{\varphi}}{1-\varphi}, \tag{20}$$

while the coefficient $\xi = 4\sqrt{\tau_0/\rho}\, c^2(1+a^2)^2/\xi_1 U_+$ can acquire for $a^2 \ll 1$ a constant value. We eliminate it by taking into account that in a viscous sublayer $\mu\partial U/\partial y = -\tau_0 y/R$, i.e., $U = \tau_0 R(1-\varphi^2)/2\mu$. If $\varphi=\varphi_0$, $U=U_0$ correspond to the sublayer boundary, then

$$\tau_0 = \frac{2\mu U_0}{R(1-\varphi_0^2)}. \tag{21}$$

Consequently, for $U_0 = mU_+$ we obtain in the sublayer

$$\frac{dF}{d\varphi} = -\frac{2m\varphi}{1-\varphi_0^2}, \quad F = m\frac{1-\varphi^2}{1-\varphi_0^2}. \tag{22}$$

For a continuous associated velocity profile at the sublayer boundary, according to Eq. (22), it is necessary to take in (20) $\xi = 2m\sqrt{\varphi_0}\,(1+\varphi_0)^{-1}$. For a quite small sublayer thickness, when $\varphi\to 1$, one can practically put $2\sqrt{\varphi_0}\,(1+\varphi_0)^{-1}=1$. Then $\xi = m$, and, according to (20), under the conditions $\varphi=\varphi_0$, $F = m$ we obtain

$$F = m\left[1 + 2(\sqrt{\varphi}-\sqrt{\varphi_0}) + \ln\frac{(1-\sqrt{\varphi})(1+\sqrt{\varphi_0})}{(1+\sqrt{\varphi})(1-\sqrt{\varphi_0})}\right], \tag{23}$$

$$\varphi_1 \leqslant \varphi \leqslant \varphi_0.$$

This solution exists in the boundary-layer region, in which $\varphi\to 1$. Therefore we write it as an expansion at $1-\varphi\to 0$, including the main terms:

$$F = m\left[1 + \frac{3}{4}(\varphi-\varphi_0) + \ln\frac{1-\varphi}{1-\varphi_0}\right], \quad \varphi_1 \leqslant \varphi \leqslant \varphi_0. \tag{24}$$

We further establish the variation in velocity in a monochromatic flow zone, in which the variation in phase is determined by relationship (12), while the vortices have size $L = a_1^2\varphi_1^2(1-\varphi_1)^{-1}R$. Taking into account (18), (19), we obtain the equation $dF/d\varphi = -\xi_2\sqrt{\varphi}\, F^2/F_1^2$, where the coefficient ξ_2 is found from the continuity condition of the associated velocity profile at the point $\varphi=\varphi_1$. Taking Eq. (20) into consideration, we find that $\xi_2 = m(1-\varphi_1)^{-1}$. Under the conditions $\varphi = 0$, $F = 1$ the velocity profile is obtained in this zone in the form

$$F = \left[1 + \frac{\beta}{F_1^2}\left(\frac{\varphi}{\varphi_1}\right)^{3/2}\right]^{-1}, \quad \beta = \frac{2m}{3(1-\varphi_1)}\varphi_1^{3/2}. \tag{25}$$

The F_1 value is determined for $\varphi=\varphi_1$ from (25): $F_1 - F_1^2 = \beta$; consequently,

$$F_1 = \frac{1}{2} + \sqrt{\frac{1}{4}-\beta}. \tag{26}$$

The choice of the sign in front of the radical in (26) is determined so that the velocity profile must become more filled upon refinement of the sublayer, i.e., at $m \to 0$. We note that (25) must be represented as $F = [1+(F_1^{-1}-1)(\varphi/\varphi_1)^{3/2}]^{-1}$ or, quite accurately, in the form of the series

$$F = 1 - \left(\frac{1}{F_1}-1\right)\left(\frac{\varphi}{\varphi_1}\right)^{3/2} + \left(\frac{1}{F_1}-1\right)^2\left(\frac{\varphi}{\varphi_1}\right)^3 - \ldots \tag{27}$$

Now, according to (13), (27) we determine the change in phase in the region $0 \leqslant \varphi \leqslant \varphi_1$:

$$b_1 = \frac{c}{a_1^2 F_1}\left(1{,}65 - \frac{0{,}9}{F_1} + \frac{0{,}25}{F_1^2}\right)\cdots \tag{28}$$

Including relations (11), (16), (28) in Eq. (10), we obtain an equation for r_1:

$$\frac{2}{\pi^2} H D^2(r_1)\exp(-2r_1^2) = \left[1 - \frac{1}{\sqrt{\pi}} D(r_1)\Phi(r_1)\right]\left[1 + \frac{2}{\pi^2} D^2(r_1)\exp(-2r_1^2)\right]^{1/2}. \tag{29}$$

In this equation

$$H = \pi\left(n - \frac{1}{4}\right)\left(1.65 - \frac{0{,}9}{F_1} + \frac{0.25}{F_1^2}\right). \tag{30}$$

The calculations show that in the practically interesting region $6 \leqslant H \leqslant 35$ Eq. (29) is well approximated by the expression

$$r_1 = \frac{2.56}{\sqrt{H}}. \tag{31}$$

We determine the relative sublayer thickness $1 - \varphi_0$. For the F_1 value known from (26) the latter is directly found from (24) for $\varphi = \varphi_1$. It can be seen that

$$1 - \varphi_0 = (1 - \varphi_1)\exp\left(\frac{1}{4} - \frac{1}{m}F_1 + \frac{3}{4}\varphi_1\right)\exp\left[\frac{3}{4}(1 - \varphi_0)\right]. \tag{32}$$

Since $3/4\ (1 - \varphi_0) \ll 1$, then $\exp[3/4(1 - \varphi_0)] = 1 + 3/4\ (1 - \varphi_0)$, in which case we obtain from (32)

$$1 - \varphi_0 = (1 - \varphi_1)\left[1 - \frac{3}{4}(1 - \varphi_1)\exp s\right]^{-1}\exp s,$$
$$s = \frac{1}{4} - \frac{1}{m}F_1 + \frac{3}{4}\varphi_1. \tag{33}$$

To describe the flow structure for a given n we now know all the required characteristics: varying the quantity β in the region $0 \leqslant \beta \leqslant 1/4$, we find F_1 from (26). Further, calculating the sequence H, r_1 by (30), (31), we determine a_0 by (11) and $a_1 = a_0 \exp(-r_1^2)$, which makes it possible to find c and then calculate φ_1 from (7) for $r = r_1$. Following that we find the relative velocity at the sublayer boundary $m = 3/2\beta(1 - \varphi_1)\varphi_1^{-3/2}$, as well as the sublayer thickness (33).

It is still necessary to establish the connection of the results obtained with the flow Reynolds number $Re = 2\rho R U_\sigma/\mu$, where U_σ is the mean flow velocity over the cross section. Comparing the usual definition $\tau_0 = \lambda\rho U_\sigma^2/8$ with (21), one immediately obtains the dependence

$$Re = \frac{32m}{q(1 - \varphi_0^2)\lambda}, \tag{34}$$

with the ratio $q = U_\sigma/U_+$. Taking into account (22), (24), (27), we find

$$q = 2\int_0^1 \varphi F d\varphi = (1 - \varphi_1^2)F_1 + m\left[\frac{1}{4}(\varphi_1 - \varphi_0) - \frac{1}{2}(1 - \varphi_1^2)\right.$$
$$\left. + \frac{1}{4}(\varphi_1^3 - \varphi_0^3)\right] + \left(1.97 - \frac{1.37}{F_1} + \frac{0.4}{F_1^2}\right)\varphi_1^2. \tag{35}$$

In the final result (35) the value of $\ln[(1 - \varphi_1)/(1 - \varphi_0)]$ is expressed in terms of F_1 by means of (24). To complete the flow calculation it is now necessary to determine the hydraulic resistance coefficient λ in terms of the remaining parameters of the problem. This brings one to turn attention to the analysis of the nonstationary structure of a thin viscous sublayer at the tube wall. This sublayer grows periodically and is destroyed as a result of fluctuations of the boundary-layer vortex, whose center is found at some distance $y = y_c$ from the tube axis. The vortex oscillation frequency is (see [4])

$$\omega = \frac{1 + a^2}{2} U_c \left|\frac{\partial b}{\partial y}\right|, \tag{36}$$

where U_c is the velocity at the center of the oscillating vortex. Since $y_c \to R$, one can put in (36) $\alpha = \alpha_0$, but the wave number $|\partial b/\partial y|$, varying more strongly, must be chosen at $y = y_c$. The vortex oscillation period $T = 2\pi/\omega$ is found, according to (36), taking account of (6):

$$T = \frac{4\pi a_0^2 y_c}{cU_c(1 + a_0^2)}. \tag{37}$$

The region in which vortex fluctuations are realized at the wall is determined by its small scale l (see above), i.e., $l = R - y_c$. This implies $R(1-\varphi_c) = \varphi_c a_0^2 R/2c\sqrt{1+a_0^2}$, where $\varphi_c = y_c/R$. Hence

$$\varphi_c = \left(1 + \frac{a_0^2}{2c\sqrt{1 + a_0^2}}\right)^{-1}. \tag{38}$$

The value of U_c is calculated from Eq. (24); more precisely, $F = mm_c$ with $\varphi = \varphi_c$,

$$m_c = \frac{U_c}{U_0} = 1 + \frac{3}{4}(\varphi_c - \varphi_0) + \ln\frac{1-\varphi_c}{1-\varphi_0}. \tag{39}$$

For a nonstationary sublayer the simplest model is that of sudden flow near the wall with velocity U_c [4]. For this flow

$$\frac{U}{U_c} = \Phi(\varepsilon), \quad \varepsilon = \frac{R-y}{2}\sqrt{\frac{\rho}{\mu t}}. \tag{40}$$

Each time at the moment of time $t = T$ there is a sublayer destruction, when its thickness has achieved the value $R(1-\varphi_0)$. In this case we have at the sublayer boundary $U = U_0$, and, according to (40):

$$\varepsilon_0 = \frac{1-\varphi_0}{4}\sqrt{\frac{cmm_c(1+a_0^2)}{2\pi q\varphi_c a_0^2}\,\mathrm{Re}}\,, \tag{41}$$

$$\frac{1}{m_c} = \Phi(\varepsilon_0). \tag{42}$$

Taking account of (34) we obtain from (41)

$$\lambda = \frac{cm^2 m_c(1+a_0^2)(1-\varphi_0)}{\pi\varphi_c(1+\varphi_0)(\varepsilon_0 q a_0)^2}. \tag{43}$$

The implicit dependence of ε_0 on m_c of the form (42) can be approximated by an explicit expression. For example, accurately up to the second sign, the probability integral can be represented in the form

$$\Phi(\varepsilon) = \left[1 - \exp\left(-\frac{4}{\pi}\varepsilon^2\right)\right]^{1/2}.$$

Then

$$\varepsilon_0 = \frac{\sqrt{\pi}}{2}\sqrt{-\ln\left(1 - \frac{1}{m_c^2}\right)}. \tag{44}$$

The dependences (34), (43), together with (38), (39), (44), determine the resistance law for flow in a tube. Thus, the solution of the problem has been completed.

Calculations were performed for the wave regions corresponding to the modes $n = 2$-9. It was discovered that the wave structure and the intensity fluctuations vary little with increasing Re. A substantial modification in flow structure occurs with transition to a new n. In this case each new wave is generated, totally determined by the Re value, while approximately at $m = 1.5$, the laminar-turbulent transition takes place. Table 1 shows the calculated wave parameters: the upper and lower parameter values correspond to a state at the initial stage of the turbulent regime, when λ starts increasing sharply ($\mathrm{Re} = \mathrm{Re}_0$), and the state with $\mathrm{Re} \simeq 1.8 \cdot 10^4$.

Vortices from a monochromatic wave, unlike vortices of an inhomogeneous boundary-layer wave, cannot, flowing in the boundary-layer wave region, remain there. This implies that the

TABLE 1. Parameters of the Wave Structure

n	φ_1	a_0	a_1	c	10^{-3} Re
3	0,319	0,531	0,243	0,46	1,43
	0,335	0,526	0,243	0,475	18
4	0,484	0,431	0,237	0,541	2,58
	0,511	0,424	0,237	0,571	17,7
5	0,576	0,375	0,230	0,578	3,85
	0,608	0,367	0,228	0,621	19,6
6	0,632	0,337	0,222	0,59	5,18
	0,664	0,331	0,220	0,642	18,3
7	0,671	0,309	0,214	0,589	6,59
	0,701	0,304	0,213	0,644	17,7
8	0,699	0,285	0,207	0,574	8,19
	0,726	0,283	0,206	0,634	17,8
9	0,732	0,264	0,199	0,582	11,8
	0,745	0,264	0,199	0,617	19,4

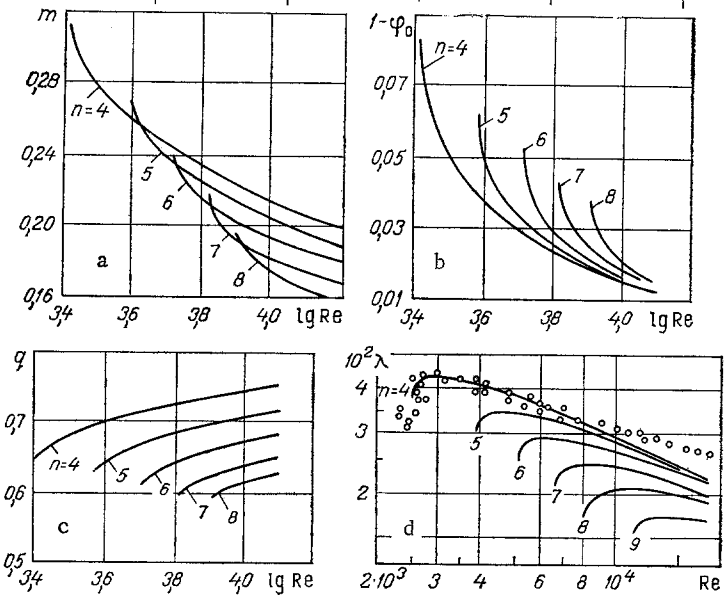

Fig. 1. Variations of relative velocities (a), sublayer thickness (b), mean velocity (c), and hydraulic resistance coefficient (d).

length of the boundary-layer transverse wave cannot be larger than the transverse size of the monochromatic zone, i.e., $1-\varphi_1 \leqslant \varphi_1$. Therefore, the existence of equilibrium turbulence regimes is possible for $\varphi_1 \geqslant 0.5$. According to Table 1, the first such regime is realized for n = 4, when φ_1 = 0.48-0.51. The corresponding Reynolds number is Re_0 = 2580, in agreement with the well-known experimental data on the least possible Re values for a developed turbulent flow in a tube. The calculated values of the Re_0 numbers for a sequence of generated transitions, starting with n = 4, are near those for which were noticed experimentally [7] an enhancement of the original random fluctuations, a strong instability, or generation of a transition. One can also recall primary facts of this nature, established by Shiller [8], who observed transitions in all cases predicted by Table 1 for n $\geqslant$ 4 (the spreads in the calculated Re_0 are insignificant). Which of the transitions is realized in a specific situation depends on the level of the original perturbations.

From the data of Table 1 one can draw conclusions on the turbulence intensity in changing the wave regimes. The calculated value $a_0 \leqslant 0.42$ of the fluctuation intensity directly near the wall, as well as the fluctuation value $a_1 \leqslant 0.24$ at the flow center are well verified experimentally. Information on the variation of separate flow parameters in the Re dependences is illustrated in Fig. 1. In particular, in Fig. 1d we show the nature of establishment of the friction law during generation of the various wave regimes, compared with experimental data. In the region Re > 10^4 the calculated λ value is lower. The reason, obviously, is the fact that the model of the sudden formulation of a nonstationary sublayer of the form (40) becomes less accurate in decreasing the fluctuation period T, when initial conditions of nonstationary viscous flow, not included in (40), begin to be important. Under these circumstances it is necessary to perform in the future further analysis of nonstationary conditions at the wall.

NOTATION

ψ, the wave function; y, radial distance from the tube axis; t, time; U, absolute value of the translational velocity; ρ, density (incompressible flow); a and b, wave amplitude and phase; ω, fluctuation frequency; R, tube radius; τ, shear stress; τ_0, shear stress at the wall; μ, viscosity coefficient; h, a "quantum" parameter, related to the circulation of large vortices; λ, hydraulic resistance coefficient; Re, Reynolds number.

LITERATURE CITED

1. Ya. A. Vagramenko, "Wave theory of turbulence," Preprint No. 137, Institute of Problems in Mechanics, Academy of Sciences of the USSR, Moscow (1980).
2. Ya. A. Vagramenko, "Application of quantum-mechanical concepts to describe jet turbulence," Kosm. Issled. Ukr., No. 15, 73-82, Naukova Dumka, Kiev (1981).
3. W. Frost and T. H. Moulden, Handbook of Turbulence, Plenum, New York (1977).
4. Ya. A. Vagramenko, "Wave properties in a transverse shear turbulent boundary layer," Inzh.-Fiz., 45, No. 3, 410-419 (1983).
5. Ya. A. Vagramenko, "Wave and regular motion in an axisymmetric turbulent jet," Gidroaeromekh. Teor. Uprugosti, No. 29, 3-15, Dnepropetrovsk (1982).
6. E. U. Repik and Yu. P. Sosedko, "Review of experimental studies in boundary-layer turbulence," in: Proc. III All-Union Seminar on Models of Continuous Media, Novosibirsk (1976), pp. 7-35.
7. V. I. Subbtin, M. Kh. Ibragimov, G. S. Taranov, and V. I. Gusakov, "Hydrodynamic features of tubes with regular artificial wall roughness," in: Turbulent Flows [in Russian], Nauka, Moscow (1977), pp. 64-69.
8. R. Shiller, Liquid Motion in Tubes [in Russian], ONTI NKTP SSSR, Moscow–Leningrad (1935).

MATHEMATICAL BOUNDARY-LAYER MODEL FOR A WIDE RANGE OF TURBULENT REYNOLDS NUMBERS

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Based on the e–ε turbulence model, a boundary-layer system of equations is proposed, describing the laminar, transition, and turbulent flow regimes.

Analysis of contemporary turbulence models [1] shows that the most promising models for describing turbulent transfer processes in boundary layers are those in which the fluctuating flow characteristics are determined as a result of simultaneous solution of the equations of turbulence intensity e and dissipation ε. For the development of turbulent flows with relatively large turbulent Reynolds numbers ($R_T = e^2/(\nu\varepsilon) > 10^3$) models of this type have been developed in detail [2] having many practical applications. In describing flows with small R_T e–ε models were first used in [3]. In this case additional corrective terms and closure functions were introduced in the equations of turbulence intensity and dissipation, but justifying several of their assumptions seemed to raise doubts [1]. Thus, for example, introduction of the additional term $-2\mu(\partial\sqrt{e}/\partial y)^2$ in the right-hand side of the e equation, due to nonvanishing of dissipation at the wall, destroys the total balance and leads to lowering of the solution stability for increasing step sizes in the longitudinal direction. There is no physical justification for the further term in the dissipation equation $2\mu_T\nu(\partial^2U/\partial y^2)^2$, which seems to affect substantially the solution results in the direct neighborhood of the streamline surface, where the gradients of the flow parameters are particularly significant.

Despite the fact that by means of the Jones–Launder model [3] it seems possible to calculate several important special cases of boundary-layer flow, such as flow with acceleration,

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